

ALGORITHM 18
RATIONAL INTERPOLATION BY CONTINUED
FRACTIONS

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comment This procedure fits to m given points (x_i, y_i) a continued fraction in the form

$$a_1 + (x - x_1) / (a_2 + (x - x_2) / (a_3 + (x - x_3) / \dots (x - x_{m-1}) / a_m) \dots)$$

It also simplifies the continued fraction to a rational function

$$(N_0 + N_1x + \dots + N_{\text{deg}}x^{\text{deg}}) / (D_0 + D_1x + \dots + D_{\text{deg}}x^{\text{deg}}),$$

where deg is at most $m \div 2$;

procedure *confr*(m, x, y, a, N, D);

real array x, y, a, N, D ; **integer** m ;

begin real aa, xx, T ; **integer** i, j, k ; **real array** $P, Q[0 : m \div 2]$

switch $sw := sw1, sw2$;

for $j := 1$ **step 1** **until** m **do**

begin $aa := y[j]$; $xx := x[j]$;

for $i := 1$ **step 1** **until** $j - 1$ **do**

$aa := (xx - x[i]) / (aa - a[i])$; $a[j] := aa$

end;

$k := 1$; $P[0] := 1$; $Q[0] := a[1]$;

mult **for** $j := 1$ **step 1** **until** $m \div 2$ **do** $P[j] := Q[j] := 0$;

for $i := 2$ **step 1** **until** m **do**

begin for $j := i \div 2$ **step** -1 **until** 1 **do**

begin $T := a[i] \times Q[j] - x[i-1] \times P[j] + P[j-1]$;

$P[j] := Q[j]$; $Q[j] := T$

end; $T := a[i] \times Q[0] - x[i-1] \times P[0]$;

$P[0] := Q[0]$; $Q[0] := T$

end; **go to** $sw[k]$;

$sw1$ **for** $j := 0$ **step 1** **until** $m \div 2$ **do** $N[j] := Q[j]$;

$k := 2$; $P[0] := 0$; $Q[0] := 1$; **go to** $mult$;

$sw2$ **for** $j := 0$ **step 1** **until** $m \div 2$ **do** $D[j] := Q[j]$

end procedure

occur. when $i = j - 1$, the difficulty is irretrievable, and the data points must be reordered.

CERTIFICATION OF ALGORITHM 18
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FRACTIONS

[R. W. Floyd, *Comm. ACM.*, Sept. 1960]

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The body of procedure *confr* was tested with the ALGOL translator system written for the LGP-30 computer by the Dartmouth College Computer Center. No syntactical errors were found in the procedure body, except for a missing semicolon after the array declaration. The translated algorithm gave satisfactory results when tested on values of $(4x + 1)/(x + 4)$ at any three of the points $x = 1, 2, 3, 4$. When all four points were used, a division overflow occurred in the statement **for** $i := 1$ **step 1** **until** $j - 1$ **do** $aa := (xx - x[i]) / (aa - a[i])$; which forms the reciprocal differences. An overflow of this type will occur whenever $y[j]$ is approximated to high accuracy by one of the continued fractions based only on the points $x[i]$, $i = 1, 2, \dots, k$ with k less than j . Unless $i = j - 1$, the difficulty may be overcome by setting aa equal to the largest real representable in the computer whenever division overflow would