ALGORITHM 37
TELESCOPE 1
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procedure Telescope 1 (N, L, eps, limit, e) ;
integer N ;
real L, eps, limit ;
array e ;

comment: Telescope 1 takes an Nth degree polynomial approximation \( \sum_{k=0}^{N} c_k x^k \) to a function which was valid to within \( \varepsilon \geq 0 \) over an interval \((0, L)\) and reduces it, if possible, to a polynomial of lower degree, valid to within limit \( > 0 \). The initial coefficients \( c_k \) are replaced by the final coefficients, and the deleted coefficients are replaced by zero. The initial eps is replaced by the final bound on the error. N is replaced by the degree of the reduced polynomial. N and eps must be variables.

This procedure computes the coefficients given in the Techniques Department of the ACM Communications, Vol. 1, No. 9, from the recursion formula

\[
 a_{k+1} = \frac{k \cdot L \cdot (2k - 1)}{2(N + k - 1) \cdot (N - k + 1)} ;
\]

begin integer k ; array d[0:N] ;

if N < 1 then go to exit ; d[N] := -e[N] ;
for k := N step - 1 until 1 do

d[k - 1] := -d[k] \times L \times k \times (k - 0.5) / ((N + k - 1) \times (N - k + 1)) ;
if eps + abs (d[0]) < limit then begin eps := eps + abs (d[0]) ;
for k := N step - 1 until 0 do e[k] := e[k] + d[k] ;
N := N - 1 ; go to start end ;

exit:

CERTIFICATION OF ALGORITHM 37
TELESCOPE 1 [K. A. Brons, Comm. ACM, Mar., 1961]
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The body of Telescope 1 was compiled and tested on the LGP 30 using the ALGOL 60 translator system developed by the Dartmouth College Computer Center. No syntactical errors were found, and the program ran satisfactorily. The 10th degree polynomial obtained by truncating the exponential series was telescoped using \( \lim_{X} = 1 \) and \( L = 1.0 \). The result was \( N = 3 \), \( eps = 0.2103506 \), and coefficients \( 0.9999865 \), \(-0.99307236 \), \(+0.4630953 \), \(-0.1025781 \). The error curve for the telescoped polynomial was compared for \( x = 0 \) and \( x = 1 \). The error extrema were bounded by \( \varepsilon \) within 0.5%. The discrepancy is within the range of input conversion and round-off error.