ALGORITHM 38
TELESCOPE 2
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procedure Telescope 2 (N, L, eps, limit, c) ; value limit, L ;
integer N ; real L, eps, limit ; array c ;

comment Telescope 2 takes an Nth degree polynomial approximation $\sum_{k=0}^{N} c_k x^k$ to a function which was valid to within $\text{eps} \geq 0$ over an interval $(-L, L)$ and reduces it, if possible, to a polynomial of lower degree, valid to within limit $>0$. The initial coefficients $c_k$ are replaced by the final coefficients, and deleted coefficients are replaced by zero. The initial $\text{eps}$ is replaced by the final bound on the error, and $N$ is replaced by the degree of the reduced polynomial. $N$ and $\text{eps}$ must be variables. This procedure computes the coefficients given in the Techniques Department of the ACM Communications, Vol. 1, No. 9, from the recursion formula

$$a_{k+1} = \frac{k \cdot L(k-1)}{(N+k-2) \cdot (N+k-2)} \cdot a_k$$

begin integer k ; real s ; array d[0 : N] ;

start:
if $N < 2$ then go to exit ; $d[N] := -c[N]$ ;
for k := N step - 2 until 2 do
$d[k-2] := -d[k] \times L \cdot 2 \times k \times (k-1) /$
$(N+k-2) \times (N+k-2) ;$
if $(N/2) - \text{enter} (N/2) = 0$ then $s := d[0]$ else $s := d[1]/N ;$
if $\text{eps} + \text{abs}(s) < \text{limit}$ then begin
$\text{eps} := \text{eps} + \text{abs}(s) ;$
for k := N step - 2 until 0 do
$c[k] := c[k] + d[k] ;$
$N := N - 1 ;$ go to start end ;
exit:

CERTIFICATION OF ALGORITHM 38
TELESCOPE 2 [K. A. BRONS, Comm. ACM, Mar., 1961]
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This procedure was tested on the CDC 160A using 160A FORTRAN. The 10th degree polynomial obtained by truncating the series expansion of $\exp (+x)$ was telescoped using $L = 1.0$ and $\text{lim} = 0.001$. The result was $N = 4$, $\text{eps} = 0.59159949 \times 3$ and coefficients $-1.0000447$, $+0.99730758$, $+0.49919675$, $+0.17734729$, $+0.043793910$. Errors were calculated for $x = -1.0, 0.62110$. The only error to exceed $\text{eps}$ was at $x = 1.0$ and was within 0.6% of $\text{eps}$. 