ALGORITHM 48
LOGARITHM OF A COMPLEX NUMBER
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procedure LOGC(a, b, c, d); value a, b, real a, b, c, d;
comment This procedure computes the number, c+di, which is
equal to \(\log(a+bi)\);
begint c := sqrt(a \times a + b \times b);
d := arctan(b/a);
c := log(c);
if a < 0 then d := d+3.1415927
end LOGC;

CERTIFICATION OF ALGORITHM 48
LOGARITHM OF A COMPLEX NUMBER (J. R. Herndon, Comm. ACM 4 (Apr., 1961), 179)
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Algorithm 48 was translated using the DECOR Algol compiler,
after certain modifications had been incorporated, and then gave
satisfactory results.

The original version will fail if \(a = 0\) when the procedure for
arctan is entered. It also assumes that \(-\pi/2 < d < 3\pi/2\), whereas the
principal value for logarithm of a complex number assumes
\(-\pi < d \leq \pi\).

Incidentally, the Algol 50 identifier for natural logarithm is \(\ln\),
not \(\log\).

The modified procedure is as follows:

procedure LOGC(a, b, c, d); value a, b; real a, b, c, d;
comment This procedure computes the number \(c+di\) which is
equal to the principal value of \(\log(a+bi)\). If \(a = 0\) then \(c\) is
put equal to \(-i\) which is used to represent \(\text{"infinity"}\);
begin integer m, n
m := sign(a); n := sign(b);
if a = 0 then begin c := -i; d := 1.5707963 \times n; go to k
end;
c := sqrt(a \times a + b \times b);
c := ln(c);
d := 1.5707963 \times (1-m) \times (1+n-n \times n) + arctan(b/a);
k: end LOGC;

REMARK ON ALGORITHM 48
LOGARITHM OF A COMPLEX NUMBER [John R. Herndon, Comm. ACM 4 (Apr. 1961)]
MARGARET L. JOHNSON AND WARD SANPREN

Considerable care must be taken in using the arctan function.
In Algorithm 48 two such difficulties are ignored. First it is
necessary, because of a resulting division by zero, to deal sepa-
rationly with the case where the real part of the complex number
is zero. Second, if the real part of the complex number is negative
and the argument of the logarithm is to have a value between
\(-\pi\) and \(\pi\) then the action depends upon the sign of the imaginary
part of the complex number. For clarity the following procedure
exhibits in sequence the alternatives:

procedure LOGC(a, b, c, d); value a, b; real a, b, c, d;
comment This procedure computes the number \(c+di\) which is
equal to \(\log(a+bi)\). It is assumed that the arctan has a value
between \(-\pi/2\) and \(\pi/2\);
begin if a > 0 then begin THETA := 0; go to SOL end;
if a <= 0 \& b <= 0 then begin THETA := 3.1415927;
go to SOL end;
if a <= 0 \& b > 0 then begin THETA := -3.1415927;
go to SOL end;
if a >= 0 \& b = 0 then begin c := d := 0;
go to RETURN end;
if a = 0 \& b > 0 then begin c := ln(b); d := 1.5707963;
go to RETURN end;
if a = 0 \& b < 0 then begin c := ln(abs(b));
d := 1.5707963; go to RETURN end;
SOL: d := arctan(b/a) + THETA;
c := sqrt(a \times a + b \times b);
c := ln(c);
RETURN: end LOGC;

REMARK ON REMARKS ON ALGORITHM 48 [B3]
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This procedure was designed to compute \(\log(a+bi)\), namely
\(c+di\), and although some very necessary precautions about its
use have already been stated, some points seem to have escaped
notice. In particular, A. P. Relph [Comm. ACM, June 1962, 347]
remarked that if \(a = 0\), then \(c\) becomes \(\text{"infinity"}\), but this only
is the case if \(b = 0\) also. Margaret L. Johnson and Ward Sangren
[Comm. ACM, July 1962, 391] conceded that \(a = b = 0\) was a special
case, but wrongly gave zero as the result. The only reasonable way
of dealing with this case is to exit to some nonlocal label and to
let the user decide whether to terminate his program or to assign
particular values to \(c\) and \(d\). The obvious values to use here are, for
\(c\), a negative number, larger than the largest which would be given
by the procedure, and possibly zero for \(d\). (In an implementation
where \(2^{-m}\) is the smallest representable nonzero number, the
largest negative value of \(c\) possible is \(-89.416\).) Finally, in the
Johnson-Sangren version of the procedure, the last conditional
statement should read

\[
\text{if } a = 0 \& b < 0 \text{ then begin } c := \ln(\text{abs}(b));
d := -1.5707963; \text{ go to END RETURN end;}
\]

the omission of the minus sign in the original being probably
typographical in origin.