ALGORITHM 52
A SET OF TEST MATRICES
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procedure TESTMATRIX (n,A); value n; integer n;
real array A;
comment This procedure places in $A$ an $n \times n$ matrix whose
inverse and eigenvalues are known. The $n$-th row and the $n$-th
column of the inverse are the set: 1, 2, 3, ... , $n$. The matrix
formed by deleting the $n$-th row and the $n$-th column of the
inverse is the identity matrix of order $n-1$;
begin
integer i, j;
real t, c, d, f;
c := t \times (t+1) \times (t+t-5)/6;
d := 1/c;
A[n, n] := -d;
for $i := 1$ step 1 until $n-1$ do
begin
f := i;
A[i, n] := d \times f;
A[n, i] := A[i, n];
A[i, i] := d \times (c-f \times f);
end
end TESTMATRIX;

CERTIFICATION OF ALGORITHM 52
A SET OF TEST MATRICES (J. R. Herndon, Comm. ACM, Apr. 1961)
H. E. GILBERT
University of California at San Diego, La Jolla, Calif.

The statement
c := t \times (t+1) \times (t+t-5)/6;
was changed to
c := n \times (n+1) \times (n+n-5)/6;
to make the inverse have the form described in the algorithm. The
algorithm was translated to FORTRAN and tested with a matrix
eigenvalue program on the CDC 6604 computer at UCSD.
The eigenvalues for the $20 \times 20$ test matrix are:
1. 1.000000
2. 1.000000
... 19. 0.01636693
20. -2.8243833

REMARK ON ALGORITHM 52
A SET OF TEST MATRICES (John R. Herndon, Comm. ACM, Apr. 1961)
G. H. DUBAY
University of St. Thomas, Houston, Tex.

In the assignment statement
c := t \times (t+1) \times (t+t-5)/6;
the $t$ is undefined. A suitable definition would be provided by
preceding (a) with $t := n$;

REMARKS ON AND CERTIFICATION OF
ALGORITHM 52
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In addition to inserting the correction indicated by H. E. Gilbert [Comm. ACM (Aug. 1961), 339] the algorithm was simplified by using the simultaneous assignment and by eliminating the local variables $t$ and $f$. The resulting algorithm is as follows:

procedure TESTMATRIX(n,A);
value n; integer n; real array A;
comment ALG. 52: This procedure places in $A$ an $n \times n$ matrix
whose inverse and eigenvalues are known. The $n$-th row and the
$n$-th column of the inverse are the set: 1, 2, 3, ... , $n$. The matrix
formed by deleting the $n$-th row and the $n$-th column of the in
verse is the identity matrix of order $n-1$;
begin integer i, j; real c, d;
c := n \times (n+1) \times (n+n-5)/6;
d := 1/c;
A[n, n] := -d;
for $i := 1$ step 1 until $n-1$ do
begin
A[i, n] := d \times i;
A[n, i] := A[i, n];
end
end TESTMATRIX;

CERTIFICATION OF ALGORITHM 52
J. S. HILLMORE

The algorithm was corrected as recommended by H. E. Gilbert
in his certification [Comm. ACM, Aug. 1961] and then successfully
run using the Elliott Algol translator on the National-Elliott 803.
The matrices so generated were used to test the matrix inversion
procedure GJR given by H. R. Schwarz in his article "An Intro-
duction to Algol." [Comm. ACM, Feb. 1962].
ADDITIONAL REMARKS ON ALGORITHM 52
A SET OF TEST MATRICES [J. R. Herndon, Comm. ACM (Apr. 1961), 180]

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From an inspection of the results of eigenvalue-finding algorithms I conclude that all but two of the eigenvalues of TEST-MATRIX are unity while the two remaining are given by the expressions \(\frac{6}{p(n(n+1))}\) and \(\frac{p}{n(5-2n)}\) where

\[ p = 3 + \sqrt{4(n-3)(n-1) + 3(n+1)}. \]

These expressions have been used for the determination of absolute errors of the eigenvalues calculated by JACOBI, Algorithm 85, and Householder Tridiagonalisation, etc. as reported below. They were also used to calculate the following table (using GIER ALGOL, with 29 significant bits):

<table>
<thead>
<tr>
<th>(n)</th>
<th>Determinant</th>
<th>Eigenvalues Differing from unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-500 000 00</td>
<td>0.224 744 87</td>
</tr>
<tr>
<td>4</td>
<td>-100 000 00</td>
<td>0.153 112 89</td>
</tr>
<tr>
<td>5</td>
<td>-0.040 000 00</td>
<td>0.115 238 08</td>
</tr>
<tr>
<td>6</td>
<td>-0.008 408 163</td>
<td>0.088 200 570</td>
</tr>
<tr>
<td>7</td>
<td>-0.001 904 762</td>
<td>0.071 428 571</td>
</tr>
<tr>
<td>8</td>
<td>-0.000 575 757 6</td>
<td>0.050 386 081</td>
</tr>
<tr>
<td>9</td>
<td>-0.000 128 205 2</td>
<td>0.050 422 549</td>
</tr>
<tr>
<td>10</td>
<td>-0.000 636 363 6</td>
<td>0.043 553 383</td>
</tr>
<tr>
<td>11</td>
<td>-0.000 673 706 8</td>
<td>0.038 097 478</td>
</tr>
<tr>
<td>12</td>
<td>-0.000 024 291 5</td>
<td>0.033 718 770</td>
</tr>
<tr>
<td>13</td>
<td>-0.000 569 858 7</td>
<td>0.030 128 103</td>
</tr>
<tr>
<td>14</td>
<td>-0.000 242 236 0</td>
<td>0.027 139 206</td>
</tr>
<tr>
<td>15</td>
<td>-0.000 000 000 0</td>
<td>0.021 619 013</td>
</tr>
<tr>
<td>16</td>
<td>-0.000 816 993 47</td>
<td>0.022 470 157</td>
</tr>
<tr>
<td>17</td>
<td>-0.000 676 132 52</td>
<td>0.020 619 002</td>
</tr>
<tr>
<td>18</td>
<td>-0.000 565 930 96</td>
<td>0.019 012 916</td>
</tr>
<tr>
<td>19</td>
<td>-0.000 478 468 90</td>
<td>0.017 606 429</td>
</tr>
<tr>
<td>20</td>
<td>-0.000 408 163 27</td>
<td>0.016 366 903</td>
</tr>
</tbody>
</table>

The figures for \(n = 20\) agree very well with the results quoted by H. E. Gilbert in his certification [Comm. ACM 4 (Aug. 1961), 339].