ALGORITHM 54
GAMMA FUNCTION FOR RANGE 1 TO 2
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real procedure Q(x); value x; real x,
comment This procedure computes \( \Gamma(x) \) for \( 1 \leq x \leq 2 \). This is
a reference procedure for the more general gamma function
procedure. \( \Gamma(z) = Q(z-1) \);
begin
\[ Q := ((0.0035868843 \times x - 0.19352782) \times x \\
+ 0.48216039) \times x - 0.75670408) \times x \\
+ 0.918205586) \times x - 0.897856941) \times x \\
+ 0.098920580) \times x - 0.57719915 \times x + 1.0)
\]
end Q;

REMARKS ON:
ALGORITHM 34 [S14]
GAMMA FUNCTION
[ M. F. Lipp, Comm. ACM 4 (Feb. 1961), 106]
ALGORITHM 54 [S14]
GAMMA FUNCTION FOR RANGE 1 TO 2
[John R. Herndon, Comm. ACM 4 (Apr. 1961), 180]
ALGORITHM 80 [S14]
RECIPIROCAL GAMMA FUNCTION OF REAL
ARGUMENT
[William Holsten, Comm. ACM 5 (Mar. 1962), 166]
ALGORITHM 221 [S14]
GAMMA FUNCTION
[Walter Gautschi, Comm. ACM 7 (Mar. 1964), 143]
ALGORITHM 291 [S14]
LOGARITHM OF GAMMA FUNCTION
[M. C. Pike and I. D. Hill, Comm. ACM 9 (Sept. 1966),
684]
M. C. Pike and I. D. Hill (Reed. 12 Jan. 1966)
Medical Research Council’s Statistical Research Unit,
University College Hospital Medical School,
London, England

Algorithms 34 and 54 both use the same Hastings approximation,
accurate to about 7 decimal places. Of these two, Algorithm
54 is to be preferred on grounds of speed.

Algorithm 80 has the following errors:
1) RGAM should be in the parameter list of RGR.
2) The lines
\( \text{if } x = 0 \text{ then begin } RGR := 0; \text{ go to EXIT end} \)
and
\( \text{if } x = 1 \text{ then begin } RGR := 1; \text{ go to EXIT end} \)
should each be followed by a semicolon.
3) The lines
\( \text{if } x = -1 \text{ then begin } RGR := 0; \text{ go to EXIT end} \)
and
\( \text{if } x > -1 \text{ then begin } RGR := RGAM(z); \text{ go to EXIT end} \)
should be separated either by else or by a semicolon and this
second line ends terminating with a semicolon.
4) The lines
BB: if \( x = -1 \) then begin \( RGR := 0; \text{ go to EXIT end} \)
and
BB: if \( x > -1 \) then begin \( RGR := RGAM(z); \text{ go to EXIT end} \)
should be in the wrong place; they should come immediately after
begin real z;

With these modifications (and the replacement of the array \( B \)
in \( RGAM \) by the obvious nested multiplication) Algorithm 80 ran
successfully on the ICT Atlas computer with the ICT Atlas
ALGOL compiler and gave answers correct to 10 significant digits.

Algorithms 80, 221 and 291 all work to an accuracy of about 10
decimal places and to evaluate the gamma function it is therefore
on grounds of speed that a choice should be made between them.
Algorithms 80 and 221 take virtually the same amount of computing
time, being twice as fast as 291 at \( x = 1 \), but this advantage
decreases steadily with increasing \( x \) so that at \( x = 7 \) the speeds are
about equal and then from this point on 291 is faster—taking only
about a third of the time at \( x = 25 \) and about a tenth of the time
at \( x = 78 \). These timings include taking the exponential of log-
gamma.

For many applications a ratio of gamma functions is required
(e.g. binomial coefficients, incomplete beta function ratio) and the
use of algorithm 291 allows such a ratio to be calculated for much
larger arguments without overflow difficulties.