ALGORITHM 55
COMPLETE ELLIPTIC INTEGRAL OF THE FIRST KIND

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```
real procedure  ELLIPTIC 1(k);  value k;  real k;
comment  This procedure computes the elliptic integral of the
  first kind K(k, π/2);
begin           real t;
                t := 1-k × k;
                ELLIPTIC 1 := (((0.032024666 × t +
                    0.054555509) × t
                  + 0.097932891) × t + 1.3862944)
                  - (((0.010944912 × t + 0.060118519) × t
                  + 0.12475074) × t + 0.5) × log (t)
end             ELLIPTIC 1;
```

CERTIFICATION OF ALGORITHM 55
COMPLETE ELLIPTIC INTEGRAL OF THE FIRST KIND [John R. Herndon, *Comm. ACM*, Apr. 1961] and
CERTIFICATION OF ALGORITHM 149
COMPLETE ELLIPTIC INTEGRAL [J. N. Merner, *Comm. ACM*, Dec. 1962]

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The bodies of Algorithm 55 and of the second procedure of Algorithm 149 were tested on the LGP-30 computer using SCALP, the Dartmouth "LOAD-AND-GO" translator for a substantial subset of ALGOL 60. The floating-point arithmetic for this translator carries 7+ significant digits.

In addition to modifications required because of the limitations of the SCALP subset, the following need correction:

In Algorithm 55:

1. The constant 0.054555509 should be 0.054544409.
2. The function *log* should be *ln*.

In procedure ELIP 2 of Algorithm 149, the statement $a := c$ should be $a := C$.

The parameters of Algorithm 149 are related to the complete elliptic integral of the first kind by: $K = a \times ELIP(a, b)$ where the parameter $m = k^2 = 1 - b/a$.

The maximum approximation error in Algorithm 55 is given by Hastings as about $0.6_{10}-6$. In addition there is the possibility of serious cancellation error in forming the complementary parameter $t = 1 - k \times k$. For k near 1, errors as great as 4 significant digits were sustained. In these regions, the complementary parameter itself is a far more satisfactory parameter.

The accuracy obtainable with Algorithm 149 is limited only by the arithmetic accuracy and the amount of effort which it is desired to expend. Six-figure accuracy was obtained with 5 applications of the arithmetic-geometric mean for $a = 1000$, $b = 2$, and with one application for $a = 500$, $b = 500$.

Neither algorithm is satisfactory for $k = 1$. The behavior for Algorithm 55 will be governed by the error exit from the logarithm procedure. Under these circumstances, Algorithm 149 goes into an endless loop. Algorithm 149 may also go into an endless loop of the terminating constant ($_{10}-8$ in the published algorithm) is too small for the arithmetic being used. For the SCALP arithmetic it was found necessary to increase this tolerance to $5.0_{10}-7$. The resulting values of the elliptic integrals were, however, accurate to within 2 in the 7th significant digit (6th decimal).

The relative efficiency of the two algorithms will depend strongly on the efficiency of the square-root and logarithm subroutines. With most systems, Algorithm 55 will provide sufficient accuracy, and will be more efficient. If a square-root operation or a highly efficient square-root subroutine is available, Algorithm 149 may well be the better method.