

ALGORITHM 56
COMPLETE ELLIPTIC INTEGRAL OF THE
SECOND KIND

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```

real procedure ELLIPTIC 2(k); value k; real k;
comment This procedure computes the elliptic integral of the
  second kind  $E(k, \pi/2)$ ;
begin      real t;
            t := 1 - k  $\times$  k;
            ELLIPTIC 2 := (((0.040905094  $\times$  t +
              0.085099193)  $\times$  t
              + 0.44479204)  $\times$  t + 1.0 - (((0.01382999  $\times$  t
              + 0.08150224)  $\times$  t + 0.24969795)  $\times$  t)  $\times$  log (t)
end      ELLIPTIC 2;

```

CERTIFICATION OF ALGORITHM 56 [S21]
COMPLETE ELLIPTIC INTEGRAL OF THE
SECOND KIND

[J. R. Herndon, *Comm. ACM* 4, (Apr. 1961), 180]

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Algorithm 56 was run on a UNIVAC 1107 using the UNIVAC 1107
ALGOL 60 compiler (dated January 25, 1965). The single-precision
floating-point arithmetic of this translator carries eight significant
digits.

Two syntactical errors were removed from the algorithm:

1. The line

$$ELLIPTIC\ 2 := ((0.040905094 \times t +$$

was changed to

$$ELLIPTIC\ 2 := ((0.040905094 \times t +$$

2. The function *log* was changed to *ln*.

In addition, the statement

$$t := 1 - k \times k$$

was removed from the algorithm and the complementary parameter itself used as input to the procedure:

```

real procedure ELLIPTIC 2 (t); value t; real t;

```

to avoid cancellation error for values of *k* near 1. [While the use of *t* as input parameter is good computationally, the name of the procedure is then slightly misleading.—J.G.H.]

Several values of the complete elliptic integral of the second kind were computed for $1 \geq t > 0$. The maximum error was found to be about $7 \cdot 10^{-7}$, compared with A. M. Legendre, *Tafeln der Elliptischen Normalintegrale*, Stuttgart, 1931. For $t = 0$ an error exit from the *ln* routine takes place.