ALGORITHM 64
QUICKSORT
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procedure quicksort (A.M,N); value M,N;
array A; integer M,N;
comment Quicksort is a very fast and convenient method of
sorting an array in the random-access store of a computer. The
entire contents of the store may be sorted, since no extra space is
required. The average number of comparisons made is \(2(M-N)\ln(N-M)\), and the average number of exchanges is one sixth this
amount. Suitable refinements of this method will be desirable for
its implementation on any actual computer;
begin integer I,J;
if M < N then begin partition (A,M,N,I,J);
quicksort (A,M,J);
quicksort (A,I,N)
end
end quicksort

CERTIFICATION OF ALGORITHMS 63, 64, 65
PARTITION, QUICKSORT, FIND [C. A. R. Hoare,
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The body of the procedure find was corrected to read:
begin integer I,J;
if M < N then begin partition (A, M, N, I, J);
if K \leq I then find (A, M, J, K)
else if J \leq K then find (A, I, N, K)
end
end

and the trio of procedures was then successfully run using the
Elliott Algosat translator on the National-Elliott 803.
The author’s estimate of \(\frac{1}{2}(N-M)\ln(N-M)\) for the number of
exchanges required to sort a random set was found to be correct.
However, the number of comparisons was generally less than
\(2(N-M)\ln(N-M)\) even without the modification mentioned
below.
The efficiency of the procedure quicksort was increased by
changing its body to read:
begin integer I,J;
if M < N−1 then begin partition (A, M, N, I, J);
quicksort (A, M, J);
quicksort (A, I, N)
end
else if N−M = 1 then begin if A[N] < A[M] then
exchange (A[M], A[N])
end
end quicksort

This alteration reduced the number of comparisons involved in
sorting a set of random numbers by 4–5 percent, and the number of
entries to the procedure partition by 25–30 percent.

CERTIFICATION OF ALGORITHMS 63, 64 AND 65,
PARTITION, QUICKSORT, AND FIND, [Comm. ACM,
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Algorithms 63, 64, and 65 have been tested using the Pegasus
ALGOL 60 Compiler developed at the De Havilland Aircraft Com-
pany Ltd., Hatfield, England.

No changes were necessary to Algorithms 63 and 64 (Partition
and Quicksort) which worked satisfactorily. However, the com-
ment that Quicksort will sort an array without the need for any
extra storage space is incorrect, as space is needed for the orga-
nization of the sequence of recursive procedure activations, or, if
implemented without using recursive procedures, for storing in-
formation which records the progress of the partitioning and
sorting.

A misprint (‘if’ for ‘else if’ on the line starting ‘else if J \leq K then
\ldots’) was corrected in Algorithm 63 (Find), but it was found that
in certain cases the sequence of recursive activations of Find
would not terminate successfully. Since Partition produces as
output two integers J and I such that elements of the array
A[M:N] which lie between A[J] and A[I] are in the positions that
they will occupy when the sorting of the array is completed, Find
should cease to make further recursive activations of itself if K
fulfills the condition \(J < K < I\).
Therefore the conditional statement in the body of Find was
changed to read
if K \leq J then find (A,M,J,K)
else if I \leq K then find (A,I,N,K)

With this change the procedure worked satisfactorily.