

ALGORITHM 90
EVALUATION OF THE FRESNEL COSINE INTEGRAL

JOHN L. CUNDIFF

Engineering Experiment Station, Georgia Institute of Technology, Atlanta, Ga.

real procedure FRESNELCOS (u) result: (frcos); **value** (u);
comment This algorithm computes the Fresnel cosine integral defined by

$$C(u) = \int_0^u \cos \frac{\pi t^2}{2} dt,$$

by evaluating the series expansion

$$C(u) = \sqrt{\frac{2x}{\pi}} \left[1 - \frac{x^2}{5 \cdot 2!} + \frac{x^4}{9 \cdot 4!} - \frac{x^6}{13 \cdot 6!} + \dots \right],$$

where $x = \pi u^2/2$. Reference: PEARCEY, T. *Table of the Fresnel Integral to Six Decimal Places*. The Syndics of the Cambridge University Press, Melbourne, Australia (1956).;

```

begin pi2 := 1.5707963; x := pi2 × (u ↑ 2); frcos := 1;
      xsqr := x ↑ 2; N := 3; term := -xsqr/2;
      frcoi := 1 + (term/5);
loop: if frcoi = .frcos then go to exit; term := -term ×
      xsqr/((2×N-2) × (2×N-3)); frcos := frcoi; frcoi :=
      frcos + term/(4×N-3); N := N + 1; go to loop;
exits: frcos := u × frcos
end FRESNELCOS;

```

REMARK ON ALGORITHMS 88, 89 AND 90
EVALUATION OF THE FRESNEL INTEGRALS

[J. L. Cundiff, *Comm. ACM*, May 1962]

MALCOLM D. GRAY

The Boeing Co., Seattle, Wash.

While coding these algorithms in FORTRAN for the IBM 7094, modifications were required (both in the formulation and in the language) before execution with any degree of speed and accuracy could be obtained. In the process it was found that the reference, *Pearcy*, contains an error in the formula for $C(u)$. This error is contained in Algorithm 88 in the formula

$$C(u) = \frac{1}{2} - \frac{\sin(x)}{\sqrt{2\pi x}} [] - \dots$$

The first minus sign above should be a plus sign.

After the necessary modifications were made, the three algorithms were found to be too large and uneconomical for our usage. A single algorithm, incorporating these three procedures, was written and is in current usage in a computer program which requires several thousand evaluations of each Fresnel integral.