## ALGORITHM 91
## CHEBYSHEV CURVE-FIT


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**procedure** CHEBFIT(m, n, X, Y); **integer** m, n; **array** X, Y;
**comment** This procedure fits the tabular function $Y(X)$ (given as m points $(X, Y)$) by a polynomial $P = \sum_{i=0}^{n} A_i X^i$. This polynomial is the best polynomial approximation of $Y(X)$ in the Chebyshev sense. Reference: STIEFEL, E. *Numerical Methods of Tchebycheff Approximation*, U. of Wisc. Press (1959), 217–232;

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begin array X[1:m], Y[1:m], T[1:m], A[0:n], AX[1:n+2],
        AY[1:n+2], AH[1:n+2], BY[1:n+2], BH[1:n+2];
    integer array IN [1:n+2]; real TMAX, H; integer i,
      j, k, imax;
    comment Initialize;
    k := (m-1)/(n+1);
    for 1 := 1 step 1 until n+1 do IN [i] := (i-1)×k + 1;
    IN[n+2] := m;
    START: comment Iteration begins;
    for i := 1 step 1 until n+2 do
        begin AX[i] := X[IN[i]];
              AY[i] := Y[IN[i]];
              AH[i] := (-1) ↑ (i-1)
        end i;
    DIFFERENCE: comment divided differences;
    for i := 2 step 1 until n+2 do
        begin
        for j := i-1 step 1 until n+2 do
        begin BY[j] := AY[j];
              BH[j] := AH[j]
        end j;
        for j := i step 1 until n+2 do
        begin AY[j] := (BY[j] -BY[j-1])/
                (AX[j] -AX[j-i+1]);
              AH[j] := (BH[j] -BH[j-1])/
                (AX[j] -AX[j-i+1])
        end j;
    end i;
    H := -AY[n+2]/AH[n+2];
    POLY: comment polynomial coefficients;
    for i := 0 step 1 until n do
        begin A[i] := AY[i] +AH[i] ×H;
              BY[i] := 0
        end i;
    BY[1] := 1; TMAX := abs(H); imax := IN[1];
    for i := 1 step 1 until n do
        begin
        for j := 0 step 1 until i-1 do
            begin
            BY[i+1-j] := BY[i+1-j] -BY[i-j] ×X[IN[i]];
            A[j] := A[j] +A[i] ×BY[i+1-j]
            end j;
        end i;
    ERROR: comment compute deviations;
    for i := 1 step 1 until m do
        begin T[i] := A[n];
            for j := 0 step 1 until n do T[i] := T[i] X[i] +A[n-j];
            T[i] := T[i] -Y[i];
            if abs(T[i]) ⩽ TMAX then go to L1;
            TMAX := abs(T[i]);
            imax := i
L1:         end i;
        for i := 1 step 1 until n+2 do
            begin
            if imax < IN[i] then go to L2;
            if imax = IN[i] then go to FIT end
            end i;
L2:     if T[imax] × T[IN[i]] < 0 then go to L3;
        IN[i] := imax;
        go to START;
L3:     if IN[1] < imax then go to L4;
        for i := 1 step 1 until n+1 do IN[n+3-i] := IN[n+2-i];
        IN[i] := imax;
        go to START;
L4:     if IN[n+2] ⩽ imax then go to L5;
        IN[i-2] := imax;
        go to START;
L5:     for i := 1 step 1 until n+1 do IN[i] := IN[i+1];
        IN[n+2] := imax;
        go to START;
FIT: end CHEBFIT
```

## CERTIFICATION OF ALGORITHM 91
## CHEBYSHEV CURVEFIT [A. Newhouse, *Comm. ACM*, May 1962]


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The CHEBFIT algorithm was translated into FORTRAN and successfully run on an IBM 1620 when the following alterations were made:

(a) 2nd line after

**comment** Initialize;

should read

**for** $i$ := 1 **step** 1 **until** $n$+1 **do** $IN[i] := (i-1) \times k + 1$;

(b) 2nd and 3rd lines after

*Poly*: **comment** polynomial coefficients;

should read

**begin** $A[i] := AY[i+1] + AH[i+1] \times H$; $BY[i+1] := 0$

## REMARKS ON ALGORITHM 91
## CHEBYSHEV CURVE FIT [A. Newhouse, *Comm. ACM 5* (May 1962), 281; *6* (April 1963), 167]


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In addition to the corrections noted by R. P. Hale [op. cit., April 1963] the following are necessary:

1. The arrays $X$, $Y$, and $A$ cannot be declared to be local within the procedure body.
2. The identifier $A$ must be included as a formal parameter.

3. It should be noted that the $X[i]$ must form a monotonic sequence.
4. **comment** cannot follow the colon following a label. This occurs in four places.
5. The **end** following **go to** *FIT* must be removed.

In addition, a large number of details can be made more concise and unnecessary operations can be eliminated. Also, it seems desirable to produce the maximum deviation as a result.

## CERTIFICATION OF ALGORITHM 91 [E2] CHEBYSHEV CURVE-FIT [Albert Newhouse *Comm. ACM 5* (May 1962), 281; *6* (April 1963), 167; *7* (May 1964), 296]

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In addition to the corrections noted by R. P. Hale [op. cit., April 1963] and P. Naur [op. cit., May 1964], the following changes are necessary:

1. The first statement should be $k := entier((m-1)/(n+1))$
2. A semi-colon should precede label $L1$.

With these changes the procedure ran successfully using Elliott 503 Algol.

Although this procedure is an implementation of a finite algorithm, roundoff errors may give rise to cyclic changes of the reference set causing the procedure to fail to terminate.

Algorithm 318 [J. Boothroyd, Chebyshev Curve-Fit (Revised), *Comm. ACM 10* (Dec. 1967), 801] avoids this cycling difficulty, uses less than half the auxiliary array space of Algorithm 91 and, on test, appears to be at least four times as fast.