ALGORITHM 91
CHEBYSHEV CURVE-FIT
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procedure CHEBFIT(m, n, X, Y); integer m, n; array X, Y;
comment This procedure fits the tabular function Y(X) (given
as m points (X, Y)) by a polynomial \( P = \sum_{i=0}^{n} A_i X^i \). This
polynomial is the best polynomial approximation of Y(X) in
the Chebyshev sense. Reference: STIFEL, E. Numerical
Methods of Chebyshev Approximation, U. of Wisc. Press (1959),
217-232;

begin array X[1:m], Y[1:m], T[1:m], A[0:n], AX[1:n+2],
AY[1:n+2], AH[1:n+2], BY[1:n+2], BH[1:n+2];
integer array IN [1:n+2]; real TMAX, H; integer i,
j, k, imax;
comment Initialize;
k := (m-1)/(n+1);
for i := 1 step 1 until n+1 do IN[i] := (i-1) \times k + 1;
IN[n+2] := m;
START: comment Iteration begins;
for i := 1 step 1 until n+2 do
begin AX[i] := X[IN[i]];
AY[i] := Y[IN[i]];
AH[i] := (-1) \uparrow (i-1)
end i;
DIFFERENCE: comment divided differences;
for i := 2 step 1 until n+2 do
begin
for j := i-1 step 1 until n+2 do
begin BY[j] := AY[j];
BH[j] := AH[j]
end j;
for j := i step 1 until n+2 do
begin
AY[j] := (BY[j] - BY[j-1]) / (AX[j] - AX[j-1] - i+1);
end j;
end i;
H := -AY[n+2]/AH[n+2];
POLY: comment polynomial coefficients;
for i := 0 step 1 until n do
begin A[i] := AY[i] \times AX[i] \times H;
BY[i] := 0
end i;
BY[1] := 1; TMAX := abs(H); imax := IN[1];
for i := 1 step 1 until n do
begin
for j := 0 step 1 until i-1 do
begin
BY[i-1-j] := BY[i-1-j] - BY[i-j] \times X[IN[i]];
end j;
end i;
ERROR: comment compute deviations;
for i := 1 step 1 until n do
begin T[i] := A[n-i];
for j := 0 step 1 until n do T[i] := T[i] \times X[i] + A[n-j];
T[i] := T[i] - Y[i];
if abs(T[i]) < TMAX then go to L1;
TMAX := abs(T[i]);
imax := i
L1: end i;
for i := 1 step 1 until n+2 do
begin
if imax < IN[i] then go to L2;
if imax = IN[i] then go to FIT end
end i;
L2: if (T[imax] \times T[IN[i]]) < 0 then go to L3;
IN[i] := imax;
go to START;
L3: if IN[i] < imax then go to L4;
for i := 1 step 1 until n+1 do
IN[n+3-i] := IN[n+2-i];
IN[i] := imax;
go to START;
L4: if IN[n+2] < imax then go to L5;
IN[i-2] := imax;
go to START;
L5: for i := 1 step 1 until n+1 do
IN[i] := IN[i+1];
IN[n+2] := imax;
go to START;
FIT: end CHEBFIT

CERTIFICATION OF ALGORITHM 91
CHEBYSHEV CURVEFIT [A. Newhouse, Comm.
ACM, May 1962]
ROBERT P. HALE
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The CHEBFIT algorithm was translated into FORTRAN and
successfully run on an IBM 1620 when the following alterations
were made:
(a) 2nd line after
\begin{verbatim}
should read
\end{verbatim}
for i := 1 step 1 until n+1 do
begin
\begin{verbatim}
end i;
\end{verbatim}
(b) 2nd and 3rd lines after
\begin{verbatim}
Poly: comment polynomial coefficients;
should read
\end{verbatim}
begin A[i] := AY[i+1] + AH[i+1] \times H; BY[i+1] := 0
\end{verbatim}

REMARKS ON ALGORITHM 91
CHEBYSHEV CURVE FIT [A. Newhouse, Comm.
ACM 5 (May 1962), 281; 6 (April 1963), 167]
PETER NAUER (Repr. 27 Sept. 1963)
Regnecentralen, Copenhagen, Denmark

In addition to the corrections noted by R. P. Hale [op. cit.,
April 1963] the following are necessary:
1. The arrays X, Y, and A cannot be declared to be local within
the procedure body.
2. The identifier A must be included as a formal parameter.
3. It should be noted that the $X[i]$ must form a monotonic sequence.

4. Comment cannot follow the colon following a label. This occurs in four places.

5. The end following go to FIT must be removed.

In addition, a large number of details can be made more concise and unnecessary operations can be eliminated. Also, it seems desirable to produce the maximum deviation as a result.

CERTIFICATION OF ALGORITHM 91 [E2]
CHEBYSHEV CURVE-FIT [Albert Newhouse Comm. ACM 5 (May 1962), 281; 6 (April 1963), 167; 7 (May 1964), 296]
J. Boothroyd (Recd. 15 May 1967 and 5 Sept. 1967)
University of Tasmania, Hobart, Tasmania, Australia.

In addition to the corrections noted by R. P. Hale [op. cit., April 1963] and P. Naur [op. cit., May 1964], the following changes are necessary:

1. The first statement should be $k := \text{enter}(m-1)/(n+1))$
2. A semi-colon should precede label $L1$.

With these changes the procedure ran successfully using Elliott 503 Algol.

Although this procedure is an implementation of a finite algorithm, roundoff errors may give rise to cyclic changes of the reference set causing the procedure to fail to terminate.

Algorithm 318 [J. Boothroyd, Chebyshev Curve-Fit (Revised), Comm. ACM 10 (Dec. 1967), 801] avoids this cycling difficulty, uses less than half the auxiliary array space of Algorithm 91 and, on test, appears to be at least four times as fast.